Donte Howell

**Game Theory In Sports**

What is Game Theory?

It is a tool used to analyze strategic behavior and trying to maximize his/her payoff of the game by anticipating the actions of the other players and responding to them correctly

History of Game Theory:

Was invented by was invented by John von Neumann and Oskar Morgenstern in 1944

Elements of Game Theory:

1) The agent:

* + Also known as the player, which refers to a person, company, or nation who have their own goals and preferences

2) The utility:

* + The amount of satisfaction, or payoff that an agent receives from the situation or an event

3) The game:

* + The situation or event that all the agents/players involved will be participating in

4) The information:

* + What a player knows about what has already happened in the game, and what can be used to come up with a good strategy

5) The representation:

* + Describes the order of play employed in the game

6) The equilibrium:

* + An outcome of or a solution to the game.

Types of games used in Game Theory for sports:

Simultaneous game:

This is the type of game where all players come up with a strategy without knowing the strategy that that the other player/players are choosing. The game is simultaneous because each player has no information about the decisions of the other player/players; therefore the decisions were made simultaneously. Simultaneous games are solved using the concept of a Nash Equilibrium.

Zero Sum Game:

When one player’s loss is equal to anther player’s gain. There the sum of the winnings and losses equal to zero

Nash Equilibrium:

When a player can receive no positive benefit from changing actions, assuming other players remain constant in their strategies. A game may have multiple Nash equilibria or none at all.

There are two different types strategies used to try to achieve a Nash Equilibrium:

* + Pure Strategy
  + Mixed Strategy

The Strategies:

Pure strategy

* + Having the complete knowledge of how a player will play a game. It determines the move a player will make for any situation he or she could face.

Mixed strategy

* + The probability of which a pure strategy will be used. This allows a player to keep an opponent guessing by randomly choosing a pure strategy. Since probabilities are continuous, there are infinitely many mixed strategies available to a player, even if the amount of pure strategies is finite.

The Formula for calculating the Mixed Strategies

|  |  |  |  |
| --- | --- | --- | --- |
| **Player 2** | | | |
| Player 1 |  | A | B |
| A | a,b | c,d |
| B | e,f | g,h |

Equation used => Let q, be the probability for player 1 and p be the probability for player 2

**q × a + (1-q) × c** = probability of player 2 doing A

**q × e + (1-q) x g** = probability of player 2 doing B

**q × a + (1-q) × c = q × e + (1-q) x g** => to find probability of maximizing player 2’s payoff

**p × a + (1-p) × e** =probability of player 1 doing A

**p × c + (1-p) x g** = probability of player 1 doing B

**p × a + (1-p) × e = p × c + (1-p) x g** => To find probability of maximizing of player 1’s payoff

Game Theory in Tennis

|  |  |  |  |
| --- | --- | --- | --- |
| **Server** | | | |
| Receiver |  | Forehand | Backhand |
| Forehand | 90,10 | 20,80 |
| Backhand | 30,70 | 60,40 |

In this example the payoff for the Receiver is the probability of saving, and the payoff for the Server is the probability of scoring,

Let’s consider the potential strategies for the Server:

If the Server always aims Backhands then the Receiver (anticipating the Backhand serve) will always move Backhands and the payoﬀs will be (60,40).

Obviously the server wants the probability to be more in his favor. So the next step would be to find the best mixed strategy for the server to have his best possible performance.

* Suppose the Server aims Forehands with q probability and Backhands with 1-q probability. Then the Receiver’s payoﬀ is:

q x 90 + (1-q) x 20 = 20 + 70q if she moves Forehands

q x 30 + (1-q) x 60 = 60 - 30q if she moves Backhands.

From these solutions the server sees that the receiver is going to want to maximize their chance of a payoff. Therefore the receiver would move:

Forehands if 20 + 70q > 60 - 30q

Backhands if 20 + 70q < 60 - 30q

Either one if 20 + 70q = 60 - 30q.

That is the Receiver’s payoﬀ is the larger of 20+70q and 60-30q

How would the server maximize his payoff?

**q × a + (1-q) × c = q × e + (1-q) x g**

Forehand: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_%

Backhand: \_\_\_\_\_\_\_\_\_\_\_\_\_\_%

Plug that percent back in….

* This solution tells the server that in order to maximize his payoﬀ the Server should aim Forehands 40% of the time and Backhands 60% of the time. In this case the Receiver’s payoﬀ will be:
  + 20 + 70 × 0.4 = 60 – 30 × 0.4 = 48%

The Result

When the Server is mixing his serves 40-60 then the Receiver’s payoﬀ will be 48%chance of saving it whether he/she moves Forehands or Backhands, or mixes between them.

Therefore the Server’s payoﬀ will be

100-48 = 52% of successfully scoring

Do it for the Receiver now using the formulas

Lets say if the receiver doesn’t mix up their strategy, then the server will move its strategy to the side more favorable them.

Suppose the Receiver moves Forehands with p probability. Then her payoﬀ is:

p × 90 + (1-p) × 30 = 30 + 60p if the Server aims Forehands

p × 20 + (1-p) × 60 = 60 - 40p if the Server aims Backhands.

The server will look to minimize the receiver's payoff so, they will aim for the smaller side:

* + Forehands if **30 + 60p** < 60 - 40p
  + Backhands if 30 + 60p > **60 - 40p**
  + Either one if 30 + 60p = 60 - 40p

To maximize the receiver’s payoff they have to set them equal to each other:

* + 30 + 60p = 60 − 40p => 100p = 30 => p = 0.3 = 30%

For the receiver to maximize their payoff they should move for a forehand 30% of the time and backhand 70% of the time.

* In this case the Receiver’s her payoﬀ will be 30 + (60 × 0.3) = 60 – (40 × 0.3) = 48. Therefore the Server’s payoﬀ will be 100-48 = 52%

Mixed Strategy Result

* Receiver: 0.3F + 0.7B
* Server: 0.4F + 0.6B

Practice Problem

|  |  |  |  |
| --- | --- | --- | --- |
| **Shooter** | | | |
| Defender |  | Jumpshot | Drive to Hoop |
| Jumpshot | 85,15 | 5,95 |
| Drive to Hoop | 45,55 | 70,30 |

Find the Mixed Strategies for the shooter and defender